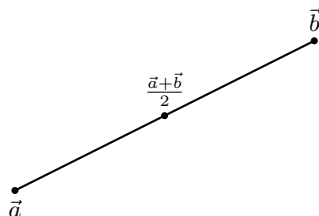


Question 8

Prove using components that the midpoint of points with position vectors $\vec{a}$ and $\vec{b}$ has position vector $(\vec{a} + \vec{b})/2$.

Question 8 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Write $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$.

Step 2 Their average is $\frac{1}{2}(\vec{a} + \vec{b}) = \left\langle \frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2} \right\rangle$.

Step 3 Each coordinate is the ordinary midpoint of the corresponding coordinates, so this is exactly the midpoint position vector. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.