

Question 10

Consider $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n^2}$.

- (a) Prove a useful power bound for $\ln n$.
- (b) Use it to test absolute convergence.
- (c) Classify the original series and explain why the AST is not needed.

Question 10 – Solution**Step 1: Dominate the logarithm.**

For $x \geq 1$, $\ln x \leq \sqrt{x}$. One proof minimizes $h(x) = \sqrt{x} - \ln x$; its minimum is $h(4) = 2 - \ln 4 > 0$.

Step 2: Test absolute convergence.

$$0 \leq \frac{\ln n}{n^2} \leq \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}.$$

The upper benchmark is a convergent p -series, so Direct Comparison proves convergence of the absolute-value series.

Step 3: Classify.

The original series converges absolutely. The AST could prove ordinary convergence after checking eventual decrease, but absolute comparison is faster and stronger.