

**Question 9**

Consider  $\sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^{-n}$ .

- (a) Find the limit of the magnitude.
- (b) Apply the nth-term test.
- (c) Explain why alternating signs cannot rescue the series.

**Question 9 – Solution**

**Step 1: Find the magnitude limit.**

The standard exponential limit gives

$$\left(1 + \frac{1}{n}\right)^n \rightarrow e, \quad \left(1 + \frac{1}{n}\right)^{-n} \rightarrow e^{-1} \neq 0.$$

**Step 2: Examine the signed terms.**

Even-indexed terms approach  $e^{-1}$  and odd-indexed terms approach  $-e^{-1}$ . Thus the terms do not approach zero.

**Step 3: Apply the nth-term test.**

A necessary condition for series convergence fails, so the series diverges. The AST is unavailable because its magnitude condition  $b_n \rightarrow 0$  fails.