

Question 8

Consider $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n)!}$.

- (a) Simplify the ratio of consecutive absolute terms.
- (b) Apply the Ratio Test and classify convergence.
- (c) Explain the effect of the doubled factorial.

Question 8 – Solution

Step 1: Form the absolute ratio.

For $b_n = n!/(2n)!$,

$$\frac{b_{n+1}}{b_n} = \frac{(n+1)!}{(2n+2)!} \frac{(2n)!}{n!} = \frac{n+1}{(2n+2)(2n+1)} = \frac{1}{2(2n+1)}.$$

Step 2: Apply the Ratio Test.

The ratio tends to $0 < 1$, so $\sum b_n$ converges. Hence the original series converges absolutely.

Step 3: Interpret.

Each step in $(2n)!$ introduces two new denominator factors but only one new numerator factor. This drives the ratio rapidly to zero.