

Question 7

Consider $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$.

- (a) Apply the Ratio Test to the absolute-value series.
- (b) Classify convergence.
- (c) Identify the exact sum using the exponential series.

Question 7 – Solution**Step 1: Test absolute values.**Let $b_n = 2^n/n!$. Then

$$\frac{b_{n+1}}{b_n} = \frac{2}{n+1} \rightarrow 0 < 1.$$

The Ratio Test proves that $\sum b_n$ converges.**Step 2: Classify.**

Therefore the signed series converges absolutely. More generally, replacing 2 by any fixed real base c gives ratio $|c|/(n+1) \rightarrow 0$.

Step 3: Find the sum.From $e^x = \sum_{n=0}^{\infty} x^n/n!$, substitute $x = -2$:

$$\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} = e^{-2}.$$