

Question 6

Consider $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$.

- (a) Verify ordinary convergence with the AST.
- (b) Test absolute convergence using the logarithmic benchmark.
- (c) Classify the series and state an alternating error estimate.

Question 6 – Solution**Step 1: Verify the AST.**

Let $b_n = 1/(n \ln n)$. For $n \geq 2$ it is positive. Its denominator is increasing, so b_n decreases, and $b_n \rightarrow 0$. Thus the alternating series converges.

Step 2: Test absolute convergence.

The absolute series is $\sum 1/(n \ln n)$. For $f(x) = 1/(x \ln x)$,

$$\int_2^b f(x) dx = \ln(\ln b) - \ln(\ln 2) \rightarrow \infty.$$

The Integral Test proves divergence.

Step 3: Classify and estimate.

The original series is conditionally convergent, with

$$|S - s_N| \leq \frac{1}{(N+1) \ln(N+1)}.$$