

Question 5

Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

- (a) Verify ordinary convergence.
- (b) Test the absolute-value series with the p -series rule.
- (c) Classify convergence and give an error bound.

Question 5 – Solution

Step 1: Apply the AST.

With $b_n = 1/\sqrt{n}$, positivity, decrease, and $b_n \rightarrow 0$ all hold. Thus the series converges.

Step 2: Test absolute convergence.

$$\sum \left| \frac{(-1)^n}{\sqrt{n}} \right| = \sum \frac{1}{n^{1/2}},$$

which diverges because $p = 1/2 \leq 1$.

Step 3: Classify and estimate.

The convergence is conditional, and

$$|S - s_N| \leq \frac{1}{\sqrt{N+1}}.$$