

Question 3

Consider $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$.

- (a) Bound the absolute value of each term.
- (b) Apply Direct Comparison to classify absolute convergence.
- (c) Explain what this implies about the original oscillatory series.

Question 3 – Solution

Step 1: Remove oscillation by an inequality.

Since $|\cos n| \leq 1$,

$$0 \leq \left| \frac{\cos n}{n^2} \right| \leq \frac{1}{n^2}.$$

Step 2: Compare absolute values.

The benchmark $\sum 1/n^2$ converges. Direct Comparison gives $\sum |\cos n|/n^2 < \infty$.

Step 3: State the strongest conclusion.

The series converges absolutely, and therefore converges ordinarily. No cancellation or special information about the values of $\cos n$ is needed.