

Question 2

Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

- (a) Prove ordinary convergence.
- (b) Test absolute convergence separately.
- (c) Classify the series, state its sum, and give an alternating error bound.

Question 2 – Solution

Step 1: Prove ordinary convergence.

For $b_n = 1/n$, we have $b_n > 0$, b_n decreases, and $b_n \rightarrow 0$. The Alternating Series Test therefore proves convergence.

Step 2: Test absolute convergence.

$$\sum \left| \frac{(-1)^n}{n} \right| = \sum \frac{1}{n},$$

which diverges. Thus the original series is conditionally convergent.

Step 3: State the value and error.

This sign convention is the negative alternating harmonic series, so

$$S = -\ln 2, \quad |S - s_N| \leq \frac{1}{N+1}.$$

Absolute convergence fails because removing the cancellation exposes the harmonic series.