

Question 1

Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{3/2}}$.

- (a) Test the absolute-value series first.
- (b) Classify the original series and identify the strongest efficient theorem.
- (c) Give an upper bound for the absolute tail after N terms.

Question 1 – Solution

Step 1: Remove the signs.

$$\sum \left| \frac{(-1)^n}{n^{3/2}} \right| = \sum \frac{1}{n^{3/2}}.$$

This is a convergent p -series because $p = 3/2 > 1$.

Step 2: Classify convergence.

Absolute convergence implies ordinary convergence, so the original series converges absolutely. Although the Alternating Series Test also proves ordinary convergence, it gives a weaker conclusion and is unnecessary here.

Step 3: Bound the absolute tail.

Since $x^{-3/2}$ is positive and decreasing,

$$\sum_{n=N+1}^{\infty} \frac{1}{n^{3/2}} \leq \int_N^{\infty} x^{-3/2} dx = \frac{2}{\sqrt{N}}.$$

This also bounds the signed remainder in magnitude.