

Question 10

Consider $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 1}$.

- (a) Use a derivative to verify decreasing magnitudes.
- (b) Apply the AST.
- (c) Test absolute convergence and classify the series.

Question 10 – Solution**Step 1: Verify monotonicity.**

Let $f(x) = x/(x^2 + 1)$. Then

$$f'(x) = \frac{1 - x^2}{(x^2 + 1)^2} \leq 0 \quad (x \geq 1).$$

Thus $b_n = n/(n^2 + 1)$ is nonincreasing and positive.

Step 2: Check the limit and apply AST.

$$b_n = \frac{1/n}{1 + 1/n^2} \longrightarrow 0.$$

Therefore the alternating series converges.

Step 3: Test absolute convergence.

$$\lim_{n \rightarrow \infty} \frac{n/(n^2 + 1)}{1/n} = \lim_{n \rightarrow \infty} \frac{n^2}{n^2 + 1} = 1.$$

By comparison with the divergent harmonic series, the absolute series diverges. Hence convergence is conditional.