

Question 9

Consider $\sum_{n=1}^{\infty} (-1)^n \sin(1/n)$.

- (a) Verify the AST hypotheses using monotonicity of sine.
- (b) Test absolute convergence by limit comparison with $1/n$.
- (c) Classify the series and state the alternating error bound.

Question 9 – Solution**Step 1: Verify the AST.**

For $n \geq 1$, $1/n \in (0, 1]$ and $b_n = \sin(1/n) > 0$. Since $1/(n+1) < 1/n$ and sine is increasing on $[0, 1]$, $b_{n+1} < b_n$. Continuity gives $b_n \rightarrow \sin 0 = 0$. Thus the series converges.

Step 2: Test absolute convergence.

$$\lim_{n \rightarrow \infty} \frac{\sin(1/n)}{1/n} = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

The harmonic series diverges, so Limit Comparison shows $\sum \sin(1/n)$ diverges. Hence convergence is conditional.

Step 3: State the error bound.

$$|S - s_N| \leq \sin\left(\frac{1}{N+1}\right) < \frac{1}{N+1}.$$