

Question 8

Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$.

- (a) Verify decrease without differentiating.
- (b) Apply the AST.
- (c) Test absolute convergence and classify the series.

Question 8 – Solution**Step 1: Verify monotonicity.**

Since $n + 2 > n + 1$ and logarithm is strictly increasing,

$$\ln(n + 2) > \ln(n + 1) > 0.$$

Taking positive reciprocals reverses the inequality, so $b_{n+1} < b_n$ for $b_n = 1/\ln(n + 1)$.

Step 2: Check the limit and apply AST.

Because $\ln(n + 1) \rightarrow \infty$, $b_n \rightarrow 0$. Thus the alternating series converges.

Step 3: Test absolute convergence.

Since $\ln(n + 1) \leq n$ for $n \geq 1$,

$$\frac{1}{\ln(n + 1)} \geq \frac{1}{n}.$$

The absolute series diverges by comparison with the harmonic series. Therefore convergence is conditional.