

Question 7

Consider $\sum_{n=1}^{\infty} (-1)^n e^{-n}$.

- (a) Verify the AST hypotheses.
- (b) Test absolute convergence using a geometric series.
- (c) Find the exact sum and classify convergence.

Question 7 – Solution**Step 1: Verify the AST.**

With $b_n = e^{-n} > 0$, we have $b_{n+1} = e^{-1}b_n < b_n$ and $b_n \rightarrow 0$. Thus the series converges.

Step 2: Test absolute convergence.

$$\sum |(-1)^n e^{-n}| = \sum (e^{-1})^n,$$

a geometric series with ratio $e^{-1} < 1$. Hence convergence is absolute.

Step 3: Find the sum.

The original series is geometric with first term $-e^{-1}$ and ratio $-e^{-1}$:

$$S = \frac{-e^{-1}}{1 + e^{-1}} = -\frac{1}{e + 1}.$$