

Question 6

Consider $\sum_{n=2}^{\infty} \frac{(-1)^n}{n + (-1)^n}$.

- (a) Explain the corrected starting index and why the usual AST magnitude is not decreasing.
- (b) Pair consecutive even and odd terms to test convergence.
- (c) Test absolute convergence and classify the series.

Question 6 – Solution

Step 1: Check the domain and AST template.

At $n = 1$ the denominator is $1 - 1 = 0$, so the series must start at 2. Magnitudes satisfy $b_{2k} = 1/(2k+1)$ and $b_{2k+1} = 1/(2k)$; hence $b_{2k+1} > b_{2k}$ infinitely often, so the standard monotonic AST does not apply.

Step 2: Pair terms.

$$a_{2k} + a_{2k+1} = \frac{1}{2k+1} - \frac{1}{2k} = -\frac{1}{2k(2k+1)}.$$

The paired series converges absolutely by comparison with $1/(4k^2)$. Since the unpaired final term tends to zero, both even and odd partial-sum subsequences approach the same limit; the original series converges.

Step 3: Test absolute convergence.

$$|a_{2k}| + |a_{2k+1}| = \frac{1}{2k+1} + \frac{1}{2k} \geq \frac{1}{2k}.$$

Thus the absolute series diverges, so convergence is conditional.