

Question 5

Consider $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$.

- (a) Use a continuous derivative to prove eventual decrease of the magnitude.
- (b) Verify the zero-limit condition and apply the AST.
- (c) Test absolute convergence and classify the series.

Question 5 – Solution**Step 1: Prove eventual decrease.**

Let $f(x) = \ln x/x$. Then

$$f'(x) = \frac{1 - \ln x}{x^2} < 0 \quad (x > e).$$

Hence $b_n = \ln n/n$ decreases for integers $n \geq 3$. Eventual decrease is sufficient; finitely many initial terms do not affect convergence.

Step 2: Check the limit.

L'Hopital's Rule gives

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0.$$

Therefore the AST proves convergence.

Step 3: Test absolute convergence.

For $n \geq 3$, $\ln n \geq 1$, so $\ln n/n \geq 1/n$. The absolute-value series diverges by comparison with the harmonic series. Thus convergence is conditional.