

Question 4

Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$.

- (a) Verify the AST hypotheses and classify absolute versus conditional convergence.
- (b) Find the smallest N for which the next-term bound b_{N+1} is strictly below 0.01, thereby guaranteeing $|S - s_N| < 0.01$.
- (c) Explain the role of the strict inequality.

Question 4 – Solution**Step 1: Verify convergence.**

With $b_n = 1/\sqrt{n}$, positivity is clear, b_n decreases, and $b_n \rightarrow 0$. The AST proves convergence. The absolute series $\sum 1/\sqrt{n}$ diverges ($p = 1/2$), so convergence is conditional.

Step 2: Apply the error estimate.

$$|S - s_N| \leq b_{N+1} = \frac{1}{\sqrt{N+1}}.$$

To guarantee an error strictly below 0.01 it is sufficient that

$$\frac{1}{\sqrt{N+1}} < \frac{1}{100} \iff N+1 > 10000.$$

Thus the smallest integer furnished by this bound is $\boxed{N = 10000}$.

Step 3: Verify strictness.

At $N = 9999$, the bound equals 0.01, not less than it. At $N = 10000$, it is $1/\sqrt{10001} < 0.01$.