

Question 3

Consider $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n}$.

- (a) Evaluate the limit of the term magnitude.
- (b) Apply the nth-term test before considering the AST.
- (c) Explain exactly which AST condition fails.

Question 3 – Solution

Step 1: Simplify the magnitude.

$$b_n = \frac{n+1}{n} = 1 + \frac{1}{n} \rightarrow 1.$$

Thus the series terms $a_n = (-1)^n b_n$ alternate near ± 1 and do not approach zero.

Step 2: Apply the nth-term test.

Every convergent series must satisfy $a_n \rightarrow 0$. Here the even subsequence tends to 1 and the odd subsequence tends to -1 , so the term sequence has no limit and the series diverges.

Step 3: Audit the AST.

Although b_n is positive and decreasing, the required condition $b_n \rightarrow 0$ fails. Alternating signs cannot compensate for terms of persistent size.