

Question 2

Consider $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$.

- (a) Verify that the Alternating Series Test applies.
- (b) Determine whether the series converges absolutely or conditionally.
- (c) Give an error bound for s_N and explain which conclusion is stronger.

Question 2 – Solution**Step 1: Apply the AST.**

For $b_n = 1/n^2$, we have $b_n > 0$, $b_{n+1} < b_n$, and $b_n \rightarrow 0$. Thus the alternating series converges.

Step 2: Test absolute convergence.

$$\sum \left| \frac{(-1)^n}{n^2} \right| = \sum \frac{1}{n^2},$$

a convergent p -series with $p = 2$. Hence the original series converges absolutely, a stronger conclusion than the AST alone provides.

Step 3: Bound the alternating error.

$$|S - s_N| \leq \frac{1}{(N+1)^2}.$$

In fact $S = -\pi^2/12$, though this exact value is not needed for classification.