

Question 1

Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$.

- (a) Verify every Alternating Series Test hypothesis.
- (b) Test absolute convergence and classify the series.
- (c) State its familiar value and give the error bound after N terms.

Question 1 – Solution

Step 1: Verify the AST hypotheses.

Write $b_n = 1/n$. Then $b_n > 0$, $b_{n+1} < b_n$, and $b_n \rightarrow 0$. Hence the series converges by the Alternating Series Test.

Step 2: Test absolute convergence.

$$\sum \left| \frac{(-1)^{n-1}}{n} \right| = \sum \frac{1}{n},$$

which diverges. Therefore convergence is conditional.

Step 3: State the value and error.

The power series for $\ln(1+x)$ at $x=1$ gives $S = \ln 2$. The Alternating Series Estimation Theorem gives

$$|S - s_N| \leq b_{N+1} = \frac{1}{N+1}.$$

Odd and even partial sums approach $\ln 2$ from opposite sides.