

**Question 7**

Consider  $\sum_{n=1}^{\infty} \frac{3^n + 1}{5^n - 2}$ .

- (a) Select a geometric benchmark from the dominant exponential terms.
- (b) Compute the Limit Comparison Test ratio.
- (c) Classify the series and identify the benchmark's common ratio.

### Question 7 – Solution

**Step 1: Select a geometric benchmark.**

The dominant quotient is  $3^n/5^n = (3/5)^n$ . Let  $b_n = (3/5)^n$ , whose common ratio  $3/5$  has absolute value less than 1; hence  $\sum b_n$  converges.

**Step 2: Compute the limiting ratio.**

$$\frac{(3^n + 1)/(5^n - 2)}{(3/5)^n} = \frac{1 + 3^{-n}}{1 - 2 \cdot 5^{-n}} \rightarrow 1.$$

The original terms and benchmark are positive for  $n \geq 1$ , and the ratio limit is finite and positive.

**Step 3: Apply Limit Comparison.**

The two series have the same convergence behavior. Since the benchmark is geometric and convergent,

$$\boxed{\sum_{n=1}^{\infty} \frac{3^n + 1}{5^n - 2} \text{ converges.}}$$