

Question 5

Consider $\sum_{n=1}^{\infty} \frac{n^3}{n^5 + 4}$.

- (a) Choose a benchmark from the dominant powers.
- (b) Compute the limiting ratio and verify all Limit Comparison Test conditions.
- (c) Classify the series and give a direct upper bound.

Question 5 – Solution

Step 1: Select the benchmark.

The leading quotient $n^3/n^5 = 1/n^2$ suggests $b_n = 1/n^2$, a convergent p -series.

Step 2: Compute the ratio limit.

$$\lim_{n \rightarrow \infty} \frac{n^3/(n^5 + 4)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{n^5}{n^5 + 4} = 1.$$

All terms are positive and the limit lies in $(0, \infty)$, so Limit Comparison proves convergence.

Step 3: Verify directly.

Since $n^5 + 4 \geq n^5$,

$$0 < \frac{n^3}{n^5 + 4} \leq \frac{1}{n^2}.$$

This upper comparison with a convergent series gives the same conclusion.