

Question 3

Consider $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^2}$ (radians).

- (a) Establish a pointwise trigonometric bound.
- (b) Compare with a standard p -series.
- (c) Classify the series and explain why no averaging argument is needed.

Question 3 – Solution

Step 1: Bound the numerator.

For every real number t , $0 \leq \sin^2 t \leq 1$. Hence for each positive integer n ,

$$0 \leq \frac{\sin^2 n}{n^2} \leq \frac{1}{n^2}.$$

Step 2: Classify the benchmark.

The series $\sum 1/n^2$ converges because it is a p -series with $p = 2 > 1$.

Step 3: Apply Direct Comparison.

The given nonnegative series is bounded term by term above by a convergent series, so it converges. No information about the average value or periodic distribution of $\sin^2 n$ is required.