

Question 1

Consider $\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n}$.

- (a) Identify the dominant power and select a standard benchmark.
- (b) Prove a direct-comparison inequality with the correct direction.
- (c) Classify the series and confirm the result by limit comparison.

State why the benchmark converges or diverges.

Question 1 – Solution

Step 1: Select the benchmark.

The denominator behaves like n^2 , so use $b_n = 1/n^2$, a convergent p -series with $p = 2$.

Step 2: Prove direct comparison.

Since $n^2 + 3n \geq n^2 > 0$,

$$0 < \frac{1}{n^2 + 3n} \leq \frac{1}{n^2}.$$

The Direct Comparison Test proves convergence.

Step 3: Verify by limit comparison.

$$\lim_{n \rightarrow \infty} \frac{1/(n^2 + 3n)}{1/n^2} = \lim_{n \rightarrow \infty} \frac{1}{1 + 3/n} = 1.$$

The finite positive limit confirms that both series share the same verdict.