

Question 9

Consider the shifted p -series

$$\sum_{n=1}^{\infty} \frac{1}{(n+3)^{5/4}}.$$

- (a) Verify the Integral Test hypotheses for $f(x) = (x+3)^{-5/4}$.
- (b) Evaluate the associated improper integral and classify the series.
- (c) Obtain two-sided bounds for R_N .
- (d) Explain why a fixed horizontal shift does not alter the underlying p -series verdict.

Question 9 – Solution**Step 1: Verify the hypotheses.**

Set $f(x) = (x + 3)^{-5/4}$ for $x \geq 1$. It is continuous and positive, and

$$f'(x) = -\frac{5}{4}(x + 3)^{-9/4} < 0,$$

so it decreases.

Step 2: Evaluate the integral.

$$\int_1^{\infty} (x + 3)^{-5/4} dx = \left[-4(x + 3)^{-1/4} \right]_1^{\infty} = \frac{4}{\sqrt{2}} < \infty.$$

Thus the shifted p -series converges. The horizontal shift changes initial values but not the tail exponent $p = 5/4 > 1$.

Step 3: Bound the remainder.

$$\boxed{\frac{4}{(N + 4)^{1/4}} \leq R_N \leq \frac{4}{(N + 3)^{1/4}}}.$$

Although convergent, the exponent is close to 1, so the error decreases slowly.