

Question 1

Consider the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}.$$

- (a) Define a continuous function f with $f(n) = 1/n^{3/2}$ and verify that it is positive, continuous, and decreasing on the required interval.
- (b) Use the Integral Test to determine whether the series converges or diverges.
- (c) If s_N is the N th partial sum and $R_N = S - s_N$, derive both an upper and a lower bound for R_N .

Show the improper-integral limit and all antiderivative work.

Question 1 – Solution**Step 1: Choose the continuous function.**

Let $f(x) = x^{-3/2}$ for $x \geq 1$. It is continuous and positive, and

$$f'(x) = -\frac{3}{2}x^{-5/2} < 0,$$

so it is decreasing. Also $f(n) = 1/n^{3/2}$.

Step 2: Evaluate the improper integral.

$$\int_1^{\infty} x^{-3/2} dx = \lim_{b \rightarrow \infty} \left[-2x^{-1/2} \right]_1^b = 2.$$

The integral converges, so the series converges by the Integral Test.

Step 3: Bound the remainder.

For a positive decreasing f ,

$$\int_{N+1}^{\infty} f(x) dx \leq R_N \leq \int_N^{\infty} f(x) dx.$$

Therefore

$$\boxed{\frac{2}{\sqrt{N+1}} \leq R_N \leq \frac{2}{\sqrt{N}}}.$$

The test proves convergence; these inequalities additionally measure the truncation error.