

**Question 7**

Evaluate  $\sum_{n=1}^{\infty} \frac{1}{(2n)^2}$ .

- (a) Factor the constant scale from the summand.
- (b) Use the Basel sum to evaluate the series.
- (c) Relate the answer to the odd-square contribution and the full series.

**Question 7 – Solution**

**Step 1: Simplify the scaled denominator.**

Since  $(2n)^2 = 4n^2$ ,

$$\frac{1}{(2n)^2} = \frac{1}{4} \frac{1}{n^2}.$$

**Step 2: Use linearity of convergent series.**

The Basel series converges, so its constant factor may be pulled outside:

$$\sum_{n=1}^{\infty} \frac{1}{(2n)^2} = \frac{1}{4} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{4} \cdot \frac{\pi^2}{6} = \boxed{\frac{\pi^2}{24}}.$$

**Step 3: Interpret the result.**

The substitution  $m = 2n$  selects precisely the even-indexed Basel terms. Their total is one-fourth of the complete sum because squaring the index contributes the scale factor  $2^2 = 4$ .

**Step 4: Check against the odd contribution.**

Adding the odd-square value  $\pi^2/8$  gives

$$\frac{\pi^2}{24} + \frac{\pi^2}{8} = \frac{\pi^2}{6},$$

recovering the full Basel sum.