

Question 5

Consider the alternating harmonic series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$.

- (a) Verify the Alternating Series Test hypotheses.
- (b) Decide whether the convergence is absolute or conditional.
- (c) Derive the value $\ln 2$ from the power series for $\ln(1+x)$ and state an error bound.

Question 5 – Solution**Step 1: Prove convergence.**

Let $b_n = 1/n$. Then $b_n > 0$, $b_{n+1} < b_n$, and $b_n \rightarrow 0$. Therefore the Alternating Series Test proves that $\sum (-1)^{n-1} b_n$ converges.

Step 2: Test absolute convergence.

Taking absolute values gives

$$\sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{n} \right| = \sum_{n=1}^{\infty} \frac{1}{n},$$

the divergent harmonic series. Hence the original series is conditionally convergent, not absolutely convergent.

Step 3: Identify the sum.

For $-1 < x \leq 1$, the endpoint-valid power-series identity is

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}.$$

Setting $x = 1$ gives

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = \ln 2.$$

Step 4: Quantify the approximation.

The Alternating Series Estimation Theorem gives $|S - s_N| \leq b_{N+1} = 1/(N+1)$. Odd partial sums lie above $\ln 2$ and even partial sums lie below it.