

Question 7

Determine whether $\sum_{n=1}^{\infty} \arctan n$ converges or diverges.

- (a) Evaluate the limit of the summand.
- (b) Apply the nth-term test and describe the growth of the partial sums.
- (c) Explain why no comparison or ratio test is needed.

Question 7 – Solution

Step 1: Apply the nth-term test first.

The summand has limit

$$\lim_{n \rightarrow \infty} \arctan n = \frac{\pi}{2} \neq 0.$$

Because the limit is not zero, the series diverges immediately by the nth-term test.

Step 2: Describe how the partial sums diverge.

Since $\arctan n \rightarrow \pi/2 > 1$, the definition of a limit guarantees an N_0 such that $\arctan n > 1$ for all $n \geq N_0$. Then for $N \geq N_0$,

$$s_N \geq s_{N_0-1} + (N - N_0 + 1),$$

so $s_N \rightarrow +\infty$ at least linearly.

Step 3: Justify why the test is decisive.

Every convergent series must have terms tending to zero: if $s_N \rightarrow S$, then $a_N = s_N - s_{N-1} \rightarrow S - S = 0$. Since this necessary condition fails, comparison, ratio, or integral tests are unnecessary.