

Question 5

Determine whether $\sum_{n=1}^{\infty} \frac{n!}{(n+1)!}$ converges or diverges.

- (a) Simplify the factorial quotient completely before selecting a test.
- (b) Express the N th partial sum using harmonic numbers.
- (c) Classify the series and explain why its factorial notation is deceptive.

Question 5 – Solution

Step 1: Simplify before selecting a test.

Use $(n+1)! = (n+1)n!$ to cancel the common factorial:

$$\frac{n!}{(n+1)!} = \frac{1}{n+1}.$$

Step 2: Rewrite the partial sums.

Thus

$$s_N = \sum_{n=1}^N \frac{1}{n+1} = \sum_{k=2}^{N+1} \frac{1}{k} = H_{N+1} - 1,$$

where H_m is the m th harmonic number.

Step 3: Apply the known harmonic behavior.

Because $H_{N+1} \rightarrow \infty$, subtracting the fixed number 1 does not change the divergence. Therefore the partial sums are unbounded and

$$\boxed{\sum_{n=1}^{\infty} \frac{n!}{(n+1)!} \text{ diverges.}}$$

Step 4: Interpret and verify.

The terms $1/(n+1)$ do approach zero, so the n th-term test alone is inconclusive. The factorial notation suggests rapid growth, but all of $n!$ cancels, leaving exactly a shifted harmonic series. Removing or shifting finitely many harmonic terms never changes convergence or divergence.