

Question 1

Determine whether $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ converges or diverges.

- (a) Apply the nth-term test and explain why its result is inconclusive.
- (b) Group terms between consecutive perfect squares and find a lower bound for each block.
- (c) Use the block estimates to classify the series.

Question 1 – Solution**Step 1: Check the necessary condition.**

Let $a_n = 1/\sqrt{n}$. Since $\sqrt{n} \rightarrow \infty$, $a_n \rightarrow 0$. This necessary condition is satisfied, so the n th-term test is inconclusive; it can prove divergence when the limit is nonzero, but it can never prove convergence.

Step 2: Choose a grouping strategy.

The terms decrease too slowly for a geometric comparison. Grouping between consecutive squares makes their sizes and the number of terms in each block easy to control. For $k \geq 1$, consider $k^2 \leq n < (k+1)^2$. There are

$$(k+1)^2 - k^2 = 2k + 1$$

terms, and every one satisfies $\sqrt{n} < k+1$, hence $1/\sqrt{n} > 1/(k+1)$. Therefore the k th block has sum greater than

$$\frac{2k+1}{k+1} > 1.$$

Step 3: Convert the block estimate into divergence.

The partial sum through $(m+1)^2 - 1$ contains m such blocks and consequently exceeds m . Since m can be arbitrarily large, the increasing sequence of partial sums is unbounded.

Step 4: State and check the conclusion.

Therefore

$$\boxed{\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \text{ diverges.}}$$

This agrees with the p -series rule: $p = 1/2 \leq 1$.