

Question 6

Consider $\sum_{n=1}^{\infty} (-1)^n$.

- (a) List the first six terms and partial sums.
- (b) Prove that the sequence of partial sums is bounded but divergent.
- (c) Apply the nth-term test and explain why bounded partial sums alone do not imply convergence.

Question 6 – Solution**Step 1: Distinguish terms from partial sums.**

The series terms are $a_n = (-1)^n$, so the first six are $-1, 1, -1, 1, -1, 1$. Adding successively gives the partial sums

$$-1, 0, -1, 0, -1, 0, \dots$$

More precisely,

$$s_{2k} = 0, \quad s_{2k-1} = -1.$$

Step 2: Test convergence of the partial sums.

The sequence (s_N) is bounded between -1 and 0 , but $s_{2k} = 0 \rightarrow 0$ while $s_{2k-1} = -1 \rightarrow -1$. Since these subsequences have different limits, (s_N) does not converge. By definition, the series therefore diverges.

Step 3: Apply the nth-term test as a check.

The terms $(-1)^n$ do not approach zero, so the nth-term test also proves divergence immediately. Indeed, if $s_N \rightarrow S$, then $a_N = s_N - s_{N-1} \rightarrow S - S = 0$. This example shows that bounded partial sums are not enough; the partial sums must approach one number.