

Question 8

Let m be a nonnegative integer. Explain algebraically why the generalized binomial series for $(1+x)^m$ terminates, and recover the ordinary finite Binomial Theorem. Illustrate with $m = 4$.

Question 8 – Solution

Step 1: Examine the generalized coefficient.

$$\binom{m}{n} = \frac{m(m-1)\cdots(m-n+1)}{n!}.$$

If $n = m + 1$, the numerator includes the factor $m - m = 0$. For every $n > m$, it still includes that zero factor. Hence

$$\binom{m}{n} = 0 \quad (n > m).$$

Step 2: Recover the finite theorem.

The formally infinite series therefore reduces to

$$(1+x)^m = \sum_{n=0}^m \binom{m}{n} x^n.$$

Because this is a polynomial identity, it is valid for every real (and complex) x ; there is no finite radius restriction.

Step 3: Illustrate with $m = 4$.

$$(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4.$$

The next coefficient is $\binom{4}{5} = 0$, so no higher powers occur.