

Question 7

Approximate $\sqrt{1.04}$ using only the constant and linear terms of the binomial series for $\sqrt{1+x}$. Use the alternating-series structure to bound the omitted correction and report the approximation.

Question 7 – Solution

Step 1: Use the binomial expansion.

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots.$$

Set $x = 0.04$.

Step 2: Keep two terms.

$$\sqrt{1.04} \approx 1 + \frac{0.04}{2} = \boxed{1.02}.$$

Step 3: Bound the error.

Beginning with the quadratic term, the omitted terms alternate and decrease in magnitude at $x = 0.04$. Therefore

$$|R| \leq \frac{(0.04)^2}{8} = \boxed{0.0002}.$$

The first correction is negative, so the linear approximation is an overestimate. More precisely,

$$1.0198 \leq \sqrt{1.04} < 1.02.$$

The actual value, $1.0198039\dots$, is consistent with this bound.