

Question 5

For $(1+x)^{1/3} = \sum_{n=0}^{\infty} c_n x^n$, derive a recurrence for c_{n+1} from c_n . Use it to compute through x^4 , then state the convergence interval.

Question 5 – Solution**Step 1: Derive the recurrence.**Since $c_n = \binom{1/3}{n}$,

$$\frac{c_{n+1}}{c_n} = \frac{(1/3) - n}{n + 1}, \quad \boxed{c_{n+1} = c_n \frac{1/3 - n}{n + 1}}, \quad c_0 = 1.$$

Step 2: Generate coefficients.

$$c_1 = \frac{1}{3}, \quad c_2 = -\frac{1}{9}, \quad c_3 = \frac{5}{81}, \quad c_4 = -\frac{10}{243}.$$

Therefore

$$\boxed{(1+x)^{1/3} = 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5x^3}{81} - \frac{10x^4}{243} + \cdots}.$$

Step 3: State convergence.

The radius is 1. Since the coefficient magnitudes are order $n^{-4/3}$, both endpoint series converge absolutely.

Conclusion.The interval is $\boxed{[-1, 1]}$.