

Question 4

Expand $(1 - 2x)^{3/2}$ through the x^4 term. Track the substitution $u = -2x$ carefully, then find the radius and endpoint behavior.

Question 4 – Solution

Step 1: Expand in an intermediate variable.

$$(1 + u)^{3/2} = 1 + \frac{3}{2}u + \frac{3}{8}u^2 - \frac{1}{16}u^3 + \frac{3}{128}u^4 + \cdots.$$

Step 2: Substitute $u = -2x$.

$$\begin{aligned}(1 - 2x)^{3/2} &= 1 + \frac{3}{2}(-2x) + \frac{3}{8}(4x^2) - \frac{1}{16}(-8x^3) + \frac{3}{128}(16x^4) + \cdots \\ &= \boxed{1 - 3x + \frac{3}{2}x^2 + \frac{1}{2}x^3 + \frac{3}{8}x^4 + \cdots}.\end{aligned}$$

Each degree- n coefficient is multiplied by $(-2)^n$.

Step 3: Determine convergence.

The condition $|u| < 1$ gives $|x| < 1/2$, so $R = 1/2$. For exponent $3/2$, coefficient magnitudes are order $n^{-5/2}$, so both endpoint series converge absolutely.

Conclusion.

The interval is $\boxed{[-1/2, 1/2]}$.