

Question 2

Treat $(1 + x)^{-1}$ as a generalized binomial series. Derive its coefficients, connect the result with a geometric series, and determine the exact interval of convergence.

Question 2 – Solution

Step 1: Compute the generalized coefficients.

For $n \geq 0$,

$$\binom{-1}{n} = \frac{(-1)(-2)\cdots(-n)}{n!} = (-1)^n.$$

Thus

$$(1+x)^{-1} = \sum_{n=0}^{\infty} (-1)^n x^n = 1 - x + x^2 - x^3 + \cdots.$$

Step 2: Connect with geometry.

This is exactly

$$\frac{1}{1 - (-x)} = \sum_{n=0}^{\infty} (-x)^n,$$

whose common ratio is $-x$. It converges when $|-x| < 1$, so $R = 1$.

Step 3: Test endpoints.

At $x = 1$, terms are $(-1)^n$; at $x = -1$, terms are 1. Neither tends to zero.

Conclusion.

The interval is $\boxed{(-1, 1)}$.