

Question 1

Use the generalized binomial theorem to find the first four nonzero terms of $(1 + x)^{1/2}$. State the radius and determine convergence at both endpoints.

Question 1 – Solution

Step 1: State the coefficient formula.

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n, \quad \binom{\alpha}{n} = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}.$$

Step 2: Substitute $\alpha = 1/2$.

$$\binom{1/2}{0} = 1, \quad \binom{1/2}{1} = \frac{1}{2}, \quad \binom{1/2}{2} = -\frac{1}{8}, \quad \binom{1/2}{3} = \frac{1}{16}.$$

Therefore

$$\boxed{\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \cdots}.$$

Step 3: Determine the interval.

The radius is $R = 1$. The coefficient magnitudes are asymptotic to a constant times $n^{-3/2}$, so the series converges absolutely at both $x = 1$ and $x = -1$.

Conclusion.

The interval is $\boxed{[-1, 1]}$.