

Question 10

Construct the quadratic Maclaurin model for the Gaussian e^{-x^2} . Determine where that polynomial is positive, and give an error bound for $|x| \leq 1$ using the first omitted term.

Question 10 – Solution

Step 1: Substitute into the exponential series.

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \cdots.$$

Keeping terms through degree 2 gives

$$\boxed{Q_2(x) = 1 - x^2}.$$

Step 2: Analyze the model's sign.

$$Q_2(x) > 0 \iff 1 - x^2 > 0 \iff |x| < 1.$$

It vanishes at $x = \pm 1$ and becomes negative for $|x| > 1$, whereas the true Gaussian is positive for every real x . This shows the model is local.

Step 3: Bound the local error.

For $|x| \leq 1$, the series in x^2 alternates with decreasing magnitudes. Therefore

$$\boxed{|e^{-x^2} - (1 - x^2)| \leq \frac{x^4}{2}}.$$

The error is order x^4 near the origin.