

Question 3

Evaluate $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$ by using a Taylor expansion. Identify why the constant and linear terms disappear and which coefficient determines the limit.

Question 3 – Solution

Step 1: Expand far enough to survive the cancellation.

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + O(x^4).$$

Step 2: Subtract the specified terms.

$$e^x - 1 - x = \frac{x^2}{2} + \frac{x^3}{6} + O(x^4).$$

The constant 1 and linear term x cancel exactly.

Step 3: Divide by x^2 .

$$\frac{e^x - 1 - x}{x^2} = \frac{1}{2} + \frac{x}{6} + O(x^2).$$

As $x \rightarrow 0$, every remaining positive-power term tends to zero.

Conclusion.

$$\boxed{\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2}}.$$

The answer is the first Taylor coefficient not removed by the numerator's subtraction.