

Question 2

Approximate $\sin(0.2)$ so that correct rounding through four decimal places is guaranteed. Use the lowest-degree odd Maclaurin polynomial whose error bound is below 5×10^{-5} .

Question 2 – Solution**Step 1: Check the linear approximation.**

For $x = 0.2$, the sine series is alternating with decreasing term magnitudes. Using only x gives

$$|R| \leq \frac{0.2^3}{3!} = 0.001333\ldots > 5 \times 10^{-5},$$

so the degree-1 polynomial does not provide the requested guarantee.

Step 2: Use the cubic polynomial.

$$P_3(x) = x - \frac{x^3}{3!}, \quad P_3(0.2) = 0.2 - \frac{0.008}{6} = \boxed{0.1986666667}.$$

Step 3: Bound the first omitted term.

$$|R| \leq \frac{0.2^5}{5!} = \frac{0.00032}{120} \approx 2.67 \times 10^{-6} < 5 \times 10^{-5}.$$

The alternating sign gives $0.1986666666 < \sin(0.2) < 0.1986693334$; both endpoints round to 0.1987. Therefore the smallest qualifying odd polynomial is cubic, and $\boxed{\sin(0.2) \approx 0.1987}$ to four decimal places.