

Question 1

Approximate $e^{0.1}$ using the cubic Maclaurin polynomial for e^x . Then use the Lagrange remainder to give a rigorous upper bound for the absolute error.

Question 1 – Solution

Step 1: Write the cubic polynomial.

$$P_3(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}.$$

Step 2: Evaluate at $x = 0.1$.

$$\begin{aligned} P_3(0.1) &= 1 + 0.1 + \frac{0.1^2}{2} + \frac{0.1^3}{6} \\ &= 1 + 0.1 + 0.005 + 0.0001666667 \\ &= \boxed{1.1051666667}. \end{aligned}$$

Step 3: Bound the error.

Taylor's theorem gives

$$R_3(0.1) = \frac{e^c(0.1)^4}{4!} \quad \text{for some } 0 < c < 0.1.$$

Because $e^c \leq e^{0.1}$,

$$|R_3(0.1)| \leq \frac{e^{0.1}(0.1)^4}{24} < 4.61 \times 10^{-6}.$$

Thus $e^{0.1}$ lies within 0.00000461 of the stated approximation.