

Question 10

Use complex exponentials to find the first four nonzero Maclaurin terms of $e^x \cos x$. Give a general coefficient formula and state the convergence domain.

Question 10 – Solution

Step 1: Encode the product as a real part.

$$e^x \cos x = \operatorname{Re} (e^x e^{ix}) = \operatorname{Re} (e^{(1+i)x}).$$

Step 2: Expand the complex exponential.

$$e^{(1+i)x} = \sum_{n=0}^{\infty} \frac{(1+i)^n x^n}{n!},$$

so the real coefficient of x^n is $\operatorname{Re}((1+i)^n)/n!$. Since $1+i = \sqrt{2}e^{i\pi/4}$,

$$e^x \cos x = \sum_{n=0}^{\infty} \frac{2^{n/2} \cos(n\pi/4)}{n!} x^n.$$

Step 3: Extract nonzero terms.

The x^2 coefficient vanishes. The first four nonzero terms are

$$\boxed{1 + x - \frac{x^3}{3} - \frac{x^4}{6}} \quad \left(\text{next term } -\frac{x^5}{30} \right).$$

The factorial denominator gives infinite radius, so the series represents the function for every real x .