

**Question 9**

Use the generalized binomial coefficients to find the first four nonzero terms of the Maclaurin series for  $\sqrt{1+x}$ . State the radius and determine convergence at both endpoints.

**Question 9 – Solution**

**Step 1: Use the binomial pattern.**

$$(1+x)^\alpha = \sum_{n=0}^{\infty} \binom{\alpha}{n} x^n, \quad \binom{\alpha}{n} = \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}.$$

With  $\alpha = 1/2$ ,

$$\binom{1/2}{0} = 1, \quad \binom{1/2}{1} = \frac{1}{2}, \quad \binom{1/2}{2} = -\frac{1}{8}, \quad \binom{1/2}{3} = \frac{1}{16}.$$

Therefore

$$\boxed{\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5x^4}{128} + \cdots}.$$

**Step 2: State radius and endpoints.**

The binomial series has  $R = 1$ . Its coefficients have magnitude asymptotic to a constant times  $n^{-3/2}$ , so it converges absolutely at both  $x = 1$  and  $x = -1$ .

**Conclusion.**

The interval is  $\boxed{[-1, 1]}$ , with sums  $\sqrt{2}$  and 0 at the endpoints.