

Question 8

Derive the Maclaurin series for $f(x) = 1/(1 - x)$ from Taylor coefficients, then connect it with the finite geometric-sum identity. Determine the exact interval of convergence.

Question 8 – Solution**Step 1: Compute Taylor coefficients.**

Repeated differentiation gives

$$f^{(n)}(x) = \frac{n!}{(1-x)^{n+1}}, \quad f^{(n)}(0) = n!.$$

Thus the Maclaurin coefficient is $f^{(n)}(0)/n! = 1$, and

$$\boxed{\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n}.$$

Step 2: Verify using finite sums.

$$S_N = 1 + x + \cdots + x^N = \frac{1 - x^{N+1}}{1 - x}.$$

If $|x| < 1$, then $x^{N+1} \rightarrow 0$, so $S_N \rightarrow 1/(1-x)$. This proves the Taylor series and geometric identity are the same construction.

Step 3: Check endpoints.

At $x = 1$ the terms are 1; at $x = -1$ they alternate between ± 1 . Both diverge by the nth-term test.

Conclusion.

$R = 1$ and the interval is $\boxed{(-1, 1)}$.