

Question 7

Derive the Maclaurin series for xe^x by multiplying a known series by x . Give two equivalent index forms, the first five nonzero terms, and the convergence domain.

Question 7 – Solution

Step 1: Multiply the exponential series.

$$xe^x = x \sum_{n=0}^{\infty} \frac{x^n}{n!} = \boxed{\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}}.$$

Multiplication by x raises every exponent by one but leaves the coefficient denominator tied to the original index.

Step 2: Reindex if powers x^k are preferred.

Set $k = n + 1$:

$$xe^x = \boxed{\sum_{k=1}^{\infty} \frac{x^k}{(k-1)!}}.$$

The first five nonzero terms are

$$x + x^2 + \frac{x^3}{2} + \frac{x^4}{6} + \frac{x^5}{24}.$$

Step 3: State convergence.

Multiplying an everywhere-convergent series by x does not change its infinite radius. Therefore the identity holds for every real x .