

Question 6

Use $\cosh x = (e^x + e^{-x})/2$ to derive its Maclaurin series. Explain why all odd powers cancel, give the degree-6 polynomial, and state the convergence domain.

Question 6 – Solution

Step 1: Expand both exponentials.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}.$$

When the series are added, odd-power coefficients cancel and even-power coefficients double.

Step 2: Divide by two.

$$\boxed{\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}}, \quad P_6(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}.$$

Step 3: State convergence.

Both exponential series converge absolutely for every real x , so their sum and scalar multiple do also. Thus the Maclaurin series for $\cosh x$ has infinite radius and represents the function on $(-\infty, \infty)$.