

Question 5

Obtain the Maclaurin series and degree-4 Taylor polynomial for e^{2x} by substitution into the series for e^u . State the convergence domain and justify it.

Question 5 – Solution

Step 1: Start with the parent series.

$$e^u = \sum_{n=0}^{\infty} \frac{u^n}{n!} \quad (u \in \mathbb{R}).$$

Set $u = 2x$:

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} = \sum_{n=0}^{\infty} \frac{2^n x^n}{n!}.$$

Step 2: Extract the requested polynomial.

$$P_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4.$$

Step 3: Check convergence.

For $a_n = (2x)^n/n!$,

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{2|x|}{n+1} \rightarrow 0$$

for every real x . Hence the radius is infinite and the series represents e^{2x} on all of $\mathbb{R}$.