

Question 4

Derive the Maclaurin series for $\log(1 + x)$ by integrating a geometric series. Give its first five nonzero terms and determine its exact interval of convergence.

Question 4 – Solution**Step 1: Expand the derivative.**For $|t| < 1$,

$$\frac{1}{1+t} = \sum_{n=0}^{\infty} (-1)^n t^n.$$

Step 2: Integrate from 0 to x .

$$\log(1+x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1} = \boxed{\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}}.$$

The first five nonzero terms are

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}.$$

Step 3: Determine endpoints.

The interior radius is 1. At $x = 1$, the alternating harmonic series converges to $\log 2$. At $x = -1$, the series becomes $-\sum 1/n$ and diverges.

Conclusion.The interval is $\boxed{(-1, 1]}$.