

Question 10

Find a Maclaurin series for $x^2/(1 + x^3)$. Identify the geometric ratio, explain the gaps between exponents, and determine the interval of convergence.

Question 10 – Solution**Step 1: Expand the reciprocal.**Use the geometric ratio $r = -x^3$:

$$\frac{1}{1+x^3} = \frac{1}{1-(-x^3)} = \sum_{n=0}^{\infty} (-1)^n x^{3n}, \quad |x^3| < 1.$$

Step 2: Multiply by x^2 .

$$\boxed{\frac{x^2}{1+x^3} = \sum_{n=0}^{\infty} (-1)^n x^{3n+2}}.$$

The exponents are 2, 5, 8, ... because multiplying by each additional ratio contributes three powers of x .

Step 3: Determine convergence.

The interior condition is $|x| < 1$, so $R = 1$. At $x = 1$, terms are $(-1)^n$; at $x = -1$, every term is 1. Neither tends to zero.

Conclusion.

The interval is $\boxed{(-1, 1)}$.